Currency Premia in Open Economies

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Abstract

We develop a two-country asset pricing model to explain countries' heterogeneous exposure to global risks. Goods production is specialized, their trade is frictionless, as are financial markets. Currency risk premia arise in equilibrium. A 'risky' currency is not simply one whose country contributes disproportionately to aggregate risk due to the size of its economy; rather, it is one that faces systematically volatile demand. How evenly global risks are shared among investors determines whether demand for a country's good—and thus currency—is more or less correlated with the aggregate state of the economy. A profitable 'carry trade' between two countries happens if the country with higher output growth faces more risky demand. Uneven risk sharing due to heterogeneity in beliefs across investors exacerbates both rate differentials and demand risk. Under some parameter conditions, an exchange rate covaries negatively with its volatility, possibly giving the impression of negative skewness in returns.

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Introduction

The uncovered interest rate parity hypothesis maintains that in expectation, returns form borrowing in a low-interest-rate currency and investing into bonds of a high-interest-rate country should be zero: high interest rate currencies should on average depreciate against currencies with low interest rates. However, empirical studies since Bilson (1981) and Fama (1984), as well as the continuing popularity of the so-called 'carry trade' among institutional investors, indicate that this hypothesis is violated for a number of currencies. Two salient examples are the Japanese Yen and the New Zealand Dollar, vis-a-vis the U.S. Dollar. Interest rates in Japan are on average approximately 200bps below U.S. rates, while rates in New Zealand are higher than U.S. rates by a similar order of magnitude. Both currencies are popular with investors looking for carry trade investments: the Japanese Yen has tended to depreciate, while the New Zealand Dollar has appreciated over sustained periods of time. This paper aims to explain the source of currency risk premia—and when they give rise to a profitable carry trade—by modeling a two-country real economy, without relying on exogenously imposed frictions to generate risk premia as an artefact of limits to international market integration.

The main results of the paper are as follows. While covered interest rate parity holds, uncovered interest rate parity (UIP) is violated because currency dynamics covary with the state of the aggregate economy and thus command a risk premium. The carry trade is profitable in equilibrium if the currency of the high interest rate country features a positive risk premium. Holding this currency is a bad hedge against aggregate risk: it tends to depreciate just when returns are particularly valuable to investors. This is the case for countries with strong output growth who face growing but uncertain demand for their good; uncertain in the sense that demand for their local good is particularly sensitive to the state of the economy. This is consistent with Lustig and Verdelhan (2007), who find that countries' currency risk premia are related to their sensitivity to U.S. consumption growth. This paper aims to shed some light on the channels through which this risk transference happens.

The setting of fully integrated goods and financial markets is in contrast to many models studying exchange rate dynamics which, more or less implicitly, rely on strict market segmentation.

Bansal and Shaliastovich (2012) focus on country-specific inflation, Verdelhan (2010) considers a habit-based explanation where investors' consumption-savings decisions affect only their local interest rates, and Colacito and Croce (2011), who look at the impact of long-run growth. In these models, countries' interest rate differentials are due to locally determined consumption risk that cannot be fully shared across borders.[1] In that case, the exchange rate translates the value of consumption across the countries, effectively capturing the time-varying cost to segmentation. Although imposing country-specific exposure to global and local factors as exogenous parameters is convenient for the purpose of exposition, it can only go so far in helping us understand the where this exogenous exposure comes from. Ready, Roussanov, and Ward (2013) make this segmentation explicit by introducing sector-specific shipping costs into an international exchange economy with specialized production.[2]

In the model of this paper is complementary in that cross-border trade is frictionless, but countries produce distinct consumption goods. The exchange rate translates the relative consumption value of output—as determined endogenously by aggregate demand and supply. Although in reality some degree of trade frictions or costs persist, a model that does not rely on the cost of autarky to drive exchange rates is of interest as the most popular 'carry trade countries', e.g. Japan, the United States, Switzerland, Australia and New Zealand, are often assumed to be among the most internationally integrated countries in terms of trading and financial frictions.

This model is more closely related to others featuring integrated multiple-country economies. It extends the results of Zapatero (1995) to the case where foreign and domestic equity markets are not redundant in equilibrium, thus allowing for a currency risk premium.[3] Martin (2011) and Hassan (2013) likewise focus on integrated markets. The presence of one unique tradable consumption good means currency risk is driven by countries' contribution to overall consumption risk, suggest-

[1]Though Barr and Priestley (2004) find that international bond markets are not fully integrated, with local market risk having significant impact on returns, Warnock and Warnock (2009) show that international capital flowing into the US government bond markets has contributed to lowering Treasury yields. This suggests that foreign investors' consumption-savings decisions do affect local interest rates.

[2]Hollifield and Uppal (1997) also consider the effect of explicit segmentation in the form of trade costs on the slope coefficient in the classic UIP regression.

[3]Pavlova and Rigobon (2007) use a similar setup, though their focus is on international stock market linkages under investment restrictions.

ing country size as a predominant factor in currency returns. Introducing multiple goods breaks up this strict link between size and risk premia, and allows relative risk sharing and preferences across countries to take on a complementary role to size.

The currency risk premium reflects the systematic component of currency dynamics, and can be decomposed into a local market risk component, and an exchange rate volatility component. This is consistent with, and indeed links, the empirical findings of Lustig et al. (2011), who tie carry trade returns to global risk factors, and the study by Menkhoff et al. (2010), that finds exchange rate volatility to explain the carry trade return Additionally, exchange rate volatility covaries with the exchange rate itself. Looking at this covariation in the time series can give the impression of skewness, which is consistent with the findings of Brunnermeier et al. (2008), Jurek (2009) and Burnside et al. (2010).

The model allows investors to hold heterogeneous beliefs about economic fundamentals. While the carry trade—a positive risk premium for high-interest currencies can arise in a homogeneous belief environment, disparities in countries' exposure to aggregate consumption risk, e.g. due to a home bias in portfolio choice, can exacerbate the systematic component of demand risk, raising the currency premium further.

Belief heterogeneity across investors also permits the introduction of a particular financial market friction that has been suggested as a source of carry trade returns. Carlson and Osler (2003), Jylha et al. (2008) and Brunnermeier et al. (2008) provide evidence that sudden liquidity requirements of large speculative investors such as hedge funds exacerbate adverse movements in the currency markets, justifying the existence of currency premia as compensation for such losses. More in line with Hollifield and Yaron (2003), who emphasize that much of the explanatory power for the puzzle comes from the real economy, in the present model the risk premium is not generated by the financial market friction. Nonetheless, the equilibrium impact of portfolio reallocation in response to sudden restrictions is consistent with the findings in that literature—sudden portfolio adjustments can lower the gains to previously set up carry trade transactions.

There are a number of other approaches in the literature aiming to provide an explanation for the forward premium puzzle.[4] Bansal (1997) and Backus et al. (2001) for example describe the impact of the term structure on currency premia and derive the characteristics of stochastic discount factors necessary in order to explain the foward premium puzzle. Backus et al. (2010) in turn consider nominal versus real effects, looking at the impact of monetary policies. These approaches have in common that they necessarily take the view of partial equilibrium in the economy. The results in this paper do not preclude such effects, but confirms that, beyond short term speculative activity or risk introduced into the economy through monetary policy, the real economy can indeed sustain such risk premia over longer horizons.

The model is set up as a dynamic endowment economy with two countries, each populated with a representative investor, and each producing a distinct good. Both agents consume both goods, which are costlessly shipped across borders. Financial markets, consisting of stock and bond markets in both countries, are complete and fully accessible to all investors.

In equilibrium, interest rates reflect investors' expectations about consumption growth rates and aggregate risk. Critically, and in contrast to single-good economies however, the two bonds allow investors to distinguish how the respective countries' output contributes to overall consumption growth and risk: the value of a claim to future output depends not only on how productive and risky its production technology is, but also how demand for this good varies over time, and how aligned such demand shifts are with supply shocks. The implications of the model are as follows.

First, UIP is violated. Interest rates are determined by the locally produced good's contribution to expected consumption growth: high growth rates—that are able to keep up with expected demand growth—imply high interest rates. A currency's risk premium reflects the comovement of demand for that country's good with the aggregate state of the economy. If demand for a country's good, and thus for the currency of the producing country, decreases precisely in bad times, this currency is a bad hedge and investors will require a positive risk premium. Such a premium is more likely in countries whose trade partners exhibit high consumption volatility, whether due to aggregate demand shocks or consumption-savings decisions.[5] Effectively, to look like a 'target currency'

[4]Lewis (1995) provides a good review of the literature.

[5]This is consistent with Jylha et al. (2008) and Ranaldo and Soderlind (2010), who find that carry returns are positively

for carry trade investors, a country must feature not just high potential output growth, but must also face systematically risky demand for its goods. This will lead to high interest rates and a positive currency risk premium, respectively. One can think of the case of Australia or New Zealand, largely commodity-driven economies: demand for their goods are dependent on how strong demand is from their large trading partners, China and the United States. If the U.S. and China carry large amounts of aggregate global risk through their investments, Australia implicitly imports this risk via their demand for Australian goods. The relative size of importer and exporter play a role, as well as the sensitivity of the importer to aggregate consumption risk.

Second, although instantaneous returns are Gaussian and exhibit no skewness, the currency premium varies over time. This can give the impression of skewness in the time series. For a carry trade transaction, skewness is detrimental if the volatility of the exchange rate rises just as the exchange rate moves against the carry trade. This negative skewness has been proposed as a source of currency risk premia, and shown to be present in exchange rates by Jurek (2009), among others. The present model delineates circumstances in which the covariance between the exchange rate and its volatility is negative—giving the impression of negative skewness: when disagreement increases (even at low levels of disagreement), risk is less evenly shared across the two countries. The wealth redistribution that results from a given shock to the economy exacerbates the initial impact on demand for goods, and lowers the expected rate of appreciation while exacerbating exchange rate volatility. Indeed, it is these same conditions that also make a high interest rate currency face a positive risk premium. It seems that a 'carry trade' currency pair is thus also more likely to exhibit exchange rate dynamics that are consistent with finding a negatively skewed distribution in the time series.

Third, introducing a leverage constraint into the model shows how sudden funding constraints impact the currency market and whether they harm carry trade positions, as has been suggested by e.g. Jylha and Suominen (2011). Almost by definition, forcing a subset of investors to reduce their leverage reduces the disparity in risk sharing across investors. This lowers the risk premium, thus reducing carry trade profits.

correlated with the risk premium on equity.

The paper proceeds as follows. Section 1 describes the model setup, section 2 establishes equilibrium, section 3 describes the source of currency risk premia, interest rate differentials, and how they are related to country and investor heterogeneity. Section 4 concludes. Proofs are in the Appendix.

1 Model

1.1 The Economy and Investor Preferences

The pure exchange economy is comprised of two countries, *home* and *foreign*, each of which specializes in the production of one good, $j = h, f$. While production is specialized, consumption is not: the two representative investors, $i = H, F$, that respectively populate the two countries derive utility from the consumption of both goods. The goods markets are frictionless: there are no transportation costs or tariffs, and both investors face the same relative price for the two goods. The output processes of the two goods are given by

$$\begin{aligned} dY_t^h &= \mu_{Y_h} Y_t^h dt + \sigma_{Y_h} Y_t^h dW_{t,\mathsf{h}}, \\ dY_t^f &= \mu_{Y_f} Y_t^f dt + \sigma_{Y_f} Y_t^f dW_{t,\mathsf{f}}. \end{aligned} \tag{1}$$

The two countries have potentially different growth rates μ_{Y_h} and μ_{Y_f}, and the production technologies are subject to uncorrelated shocks: σ_{Y_h} and σ_{Y_f} characterize the sensitivity of output to these fundamental shocks.[6] The uncorrelated Brownian motions $dW_{t,\mathsf{h}}$ and $dW_{t,\mathsf{f}}$ represent the *home* and *foreign* countries' respectively local production shocks.[7]

Investor i maximizes expected utility $E\left[\int_0^T u_i\left(C_{it}^{\mathsf{h}}, C_{it}^{\mathsf{f}}\right) dt\right]$, subject to his budget constraint. Utility functions of both investors are separable and additive over the two goods in the economy,

[6]Time-subscripts on parameters μ and σ above are suppressed, in the interest of parsimony of notation. As a special case, the production processes can be assumed to follow geometric Brownian motions, but the model goes through as long as the parameters are assumed to be adapted processes.

[7]Both of these naturally contribute to aggregate risk in the economy. One could formulate the economy in an alternative way, splitting the production risk into exposure to aggregate and idiosyncratic risk, without changing the nature of the results.

but both investors have a bias: they both have a preference for their respectively local good.

$$u_H\left(C^{\mathsf{h}}_{Ht}, C^{\mathsf{f}}_{Ht}\right) = \alpha^H_t \log C^{\mathsf{h}}_{Ht} + (1-\alpha^H_t)\log C^{\mathsf{f}}_{Ht}, \tag{2}$$

$$u_F\left(C^{\mathsf{h}}_{Ft}, C^{\mathsf{f}}_{Ft}\right) = (1-\alpha^F)\log C^{\mathsf{h}}_{Ft} + \alpha^F \log C^{\mathsf{f}}_{Ft}. \tag{3}$$

The preference parameters α^H_t and $\alpha^F \in [0.5, 1]$ capture this home bias in consumption.

Beyond production technology shocks, the economy is also affected by demand shocks. The relative preference parameter α^H_t varies over time, and follows a martingale uncorrelated with production shocks (see http://online.wsj.com/articles/demand-for-sand-takes-off-thanks-to-fracking-1407193760, demand for sand higher due to fracking):

$$d\alpha^H_t = \sigma_{t,\alpha} dW_{t,\alpha}. \tag{4}$$

While one can imagine certain demand shocks to be correlated with production shocks, for simplicity in this model we abstract from this and focus on unrelated demand shifts, such as for example the impact of inclement weather forecasts on the demand for heating oil or timber.[8] A home bias in consumption patterns is consistently found empirically, often attributed to the non-tradability of certain goods (notably services) and familiarity. For parsimony, in this model the bias and its time variation is not modeled in detail but exogenously imposed in (4).[9]

1.2 Financial Markets

The capital markets consist of two positive net supply stocks as well as two zero net supply bonds, which are accessible to both investors. The risky claim to the future output of *home* good Y^h will be referred to as the *home stock* S^h_t, and S^f_t is the claim to future output of *foreign* good Y^f. The

[8]To ensure that α^H_t remains above 0.5, $\sigma_{t,\alpha}$ must vary over time. For example, H's preferences may be related to an underlying state variable x_t taking the form $\alpha^H_t = 1 - 0.5/(1+x_t)$. This ensures α^H_t remains within the appropriate bounds if x_t follows an Ito process. Another example of an admissible process is $\alpha^H_t = E[\alpha^H_T|\mathcal{F}_t]$, where the terminal value of the preference parameter is a random variable between 0.5 and 1.

[9]This approach to modeling demand shocks is consistent with Dornbusch, Fischer, and Samuelson (1977), who noted the importance of allowing for demand shifts in an international model of multiple-good economies.

place of listing is immaterial—there are no differential transaction costs of trading stocks for the two agents. The 'geography' of the stocks is determined simply by the good they are a claim to. The two stocks follow the dynamics

$$dS_t^h = \mu_t^{S_h} S_t^h dt + \vec{\sigma}_t^{S_h} S_t^h d\vec{W}_t, \tag{5}$$

$$dS_t^f = \mu_t^{S_f} S_t^f dt + \vec{\sigma}_t^{S_f} S_t^f d\vec{W}_t. \tag{6}$$

Parameters $\mu_t^{S_j}$ and $\vec{\sigma}_t^{S_j}$ are determined in equilibrium. $\vec{\sigma}_t^{S_j}$ is the three-dimensional vector of stock S_t^j's sensitivities with respect to the mutually uncorrelated supply and demand shocks $d\vec{W}_t = \left(dW_{t,\mathsf{h}}, dW_{t,\mathsf{f}}, dW_{t,\alpha}\right)^\top$.

Bonds are traded in both countries, creating 'locally' riskless assets that effectively provide a forward contract on one unit of future local production.

$$db_t^h = r_t^h b_t^h dt \qquad \text{in terms of good } Y_t^h, \tag{7}$$

$$db_t^f = r_t^f b_t^f dt \qquad \text{in terms of good } Y_t^f. \tag{8}$$

While default is ruled out, the real exchange rate between *home* and *foreign* countries makes (at least) one bond's payoffs potentially risky in terms of consumption choices. The exchange rate—or relative price of the country-specific goods—reflects the respective consumption value of goods to investors. In equilibrium, this varies over time, the exchange rate $\bar{p}_t = p_t^h / p_t^f$ is determined by supply of and demand for each of the goods at time t.

Without loss of generality, the *foreign* good is set as the numeraire good in the remainder of the paper, rendering $p_t^f = 1$ and thus B_t^f the instantaneously riskless asset in this economy. The *home* bond can also be expressed in units of *foreign* 'currency': $dB_t^h = d(p_t^h b_t^h)$. dB_t^h provides risky returns in terms of numeraire 'currency', though payoff is fixed in terms of local 'currency'.

1.3 Information Structure

Uncertainty in the economy is characterized by the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$. However, investors can potentially hold different beliefs about the expected growth rates of the two economies, *home* and *foreign*.[10] Economic output and demand shocks are observable to all investors; investors' incomplete filtration $\{\mathcal{F}_t^{Y_{h,f}}\}$ is generated by processes Y_t^h and Y_t^f.

The relation between the two rational investors' beliefs is determined by observational equivalence.

$$\begin{aligned} dY_t^j &= \mu_{Y_j} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\mathrm{j}} \\ &= m_{Y_j,t}^{(\mathrm{H})} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\mathrm{j}}^{(\mathrm{H})} \\ &= m_{Y_j,t}^{(\mathrm{F})} Y_t^j dt + \sigma_{Y_j} Y_t^j dW_{t,\mathrm{j}}^{(\mathrm{F})}. \text{ for j= h,f} \end{aligned} \tag{9}$$

The relationship between investors' perceptions about the three uncorrelated sources of risk in the economy is given by

$$d\vec{W}_t^{(\mathrm{F})} = d\vec{W}_t^{(\mathrm{H})} - \Delta\vec{m}_{t,Y} dt \qquad \text{where} \qquad \Delta\vec{m}_{t,Y} = \vec{\Sigma}^{-1}(\vec{m}_t^{(\mathrm{F})} - \vec{m}_t^{(\mathrm{H})}) \tag{10}$$

where Σ is the 3×3 diffusion matrix of the economy's fundamental processes, output and demand:

$$\begin{pmatrix} dW_{t,\mathrm{h}}^{(\mathrm{F})} \\ dW_{t,\mathrm{f}}^{(\mathrm{F})} \\ dW_{t,\alpha}^{(\mathrm{F})} \end{pmatrix} = \begin{pmatrix} dW_{t,\mathrm{h}}^{(\mathrm{H})} \\ dW_{t,\mathrm{f}}^{(\mathrm{H})} \\ dW_{t,\alpha}^{(\mathrm{H})} \end{pmatrix} - \begin{pmatrix} \sigma_{Y_h} & 0 & 0 \\ 0 & \sigma_{Y_f} & 0 \\ 0 & 0 & \sigma_{t,\alpha} \end{pmatrix}^{-1} \begin{pmatrix} m_{Y_h,t}^{(\mathrm{F})} - m_{Y_h,t}^{(\mathrm{H})} \\ m_{Y_f,t}^{(\mathrm{F})} - m_{Y_f,t}^{(\mathrm{H})} \\ 0 \end{pmatrix} dt. \tag{11}$$

The elements of $\Delta\vec{m}_{t,Y}$ capture the relative optimism of investor *F* compared to investor *H* regarding the two countries' growth rates. As demand shocks are observable and follow a martingale, there is no room for rational disagreement, the last element of $\Delta\vec{m}_{t,Y}$ is equal to zero. A 'home

[10] The volatility components of economic output are known by investors. Quadratic variation allows them to draw exact inferences about the diffusion terms of dY_t^h and dY_t^f, as well as demand shocks $d\alpha_t^H$. This form of 'agreeing to disagree' was noted by Morris (1995) and has been widely used in the asset pricing literature. See, e.g. Basak (2005).

bias' about investment opportunities would be captured by a negative first element of $\Delta \vec{m}_{t,Y}$, and a second positive element.[11]

2 Equilibrium

Aggregating both investors into one representative agent

$$U(C_H, C_F) = u_H\left(C_{Ht}^h, C_{Ht}^f\right) + \lambda_t u_F\left(C_{Ft}^h, C_{Ft}^f\right), \tag{12}$$

λ_t captures the weight of investor F relative to investor H (whose weight is normalized to 1) in a competitive equilibrium. λ_t reflects investors' initial endowments and any potential differences in investors' state price densities arising from disagreement: $\lambda_t = \psi_H \xi_t^H / \psi_F \xi_t^F$. Both investors' budget constraints must be satisfied:

$$dX_t^i = X_t^i \left[\sum_{j=h}^{f} \pi_{it}^{S_j}(dS_t^j + p_t^j Y_t^j dt)/S_t^j + \sum_{j=h}^{f} \pi_{it}^{B_j} dB_t^j / B_t^j\right] - \sum_{j=h}^{f} p_t^j C_{it}^j dt \qquad \text{for } i = H, F \tag{13}$$

where $X_t^i \geq 0$ is agent i's wealth, $\pi_{it}^{S_j}$ is the fraction of wealth investor i chooses to invest in stock S_t^j, and $\pi_{it}^{B_j}$ the fraction invested into bond B_t^j.

Under belief heterogeneity, even a complete market will reflect differences across agents' state price densities, due to differences in perception that are then reflected in portfolio holdings and market prices of assets. While such differences are not necessary for an equilibrium with exchange rate risk premia to emerge in this setting, allowing for investor heterogneity allows us to anlayse the impact of cross-border heterogeneity in investment decisions on currency dynamics. Setting differences in beliefs $\Delta \vec{m}_{t,Y}$ to zero is then simply a special case. Using (10), state price densities

[11] Time-subscripts in $m_{Y_h,t}^{(i)}$ and $m_{Y_f,t}^{(i)}$ reflect that beliefs could be subject to learning, as investors update their beliefs using observed signals. The specifics of learning dynamics are not critical to establishing equilibrium in this model, as long as the process of investor disagreement can be assumed to be bounded.

follow

$$
\begin{aligned}
d\xi_t^H &= -r_t\xi_t^H dt - \vec{\kappa}_t^{H\top}\xi_t^H d\vec{W}_t^{(H)}, && (14)\\
d\xi_t^F &= -r_t\xi_t^F dt - \vec{\kappa}_t^{F\top}\xi_t^F d\vec{W}_t^{(F)}. && (15)
\end{aligned}
$$

where $\vec{\kappa}_t^i = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(i)} - r_t\mathbf{1}\right)$ is investor i's market price of risk based on his beliefs about both countries' fundamental growth rates, and by observational equivalence $\Delta\vec{\kappa}_t = \Delta\vec{m}_{t,Y}$ must hold.

Proposition 1 gives equilibrium consumption, portfolio holdings and stock prices. Though production risk of the *home* and *foreign* economies is uncorrelated, stock and bond markets are related through equilibrium prices of the countries' output, arising from investors' consumption and investment choices. $\bar{p}_t = p_t^h/p_t^f$ reflects the terms of trade: relative demand for and supply of countries' output. This is the real exchange rate between the countries and is the conductor for fundamental shocks to propagate from the goods into the financial markets.

Proposition 1. *Market-clearing consumption shares of good $i = h, f$ are characterized by s_i^F for agent F, and $(1 - s_i^F)$ for investor H.*

$$
\begin{aligned}
C_{Ft}^h &= \frac{\lambda_t\left(1-\alpha^F\right)}{\alpha_t^H + \left(1-\alpha^F\right)\lambda_t}Y_t^h = s_h^F Y_t^h; \qquad C_{Ht}^h = (1-s_h^F)Y_t^h\\
C_{Ft}^f &= \frac{\lambda_t\alpha^F}{1-\alpha_t^H + \alpha^F\lambda_t}Y_t^f = s_f^F Y_t^f; \qquad C_{Ht}^f = (1-s_f^F)Y_t^f && (16)
\end{aligned}
$$

Taking good Y_t^f to be the numeraire, equilibrium stock and bond prices in the home country are functions of the relative price of the local good, $\bar{p}_t = p_t^h/p_t^f = \xi_t^h/\xi_t^f$.

$$
\begin{aligned}
S_t^h &= \bar{p}_t Y_t^h (T-t), && (17)\\
B_t^h &= \bar{p}_t b_t^h, && (18)\\
S_t^f &= Y_t^f (T-t), && (19)\\
B_t^f &= b_t^f \qquad\qquad \forall t && (20)\\
\textit{where}\quad \bar{p}_t &= \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{(1-\alpha_t^H) + \alpha^F\lambda_t}\frac{Y_t^f}{Y_t^h}. && (21)
\end{aligned}
$$

λ_t follows dynamics $d\lambda_t = \lambda_t \Delta\vec{\kappa}_t^\top d\vec{W}_t^{(H)}$, where $\Delta\vec{\kappa}_t^\top = \left[\Delta m_t^h, \Delta m_t^f, 0\right]$ captures differences in investors' market prices of home, foreign, and demand risk. For the special case of homogeneous beliefs, λ_t is constant λ, and determined exclusively by initial endowment of the two investors. Portfolio weights of investors $i = H, F$ are

$$\pi_{it} \quad = \quad (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \left(\vec{m}_{S,t}^{(i)} - r_t \mathbf{1}\right) \tag{22}$$

where $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^\top$ are the fractions of i's wealth invested into the two stocks and the home *bond. The budget constraint implies $\pi_{i,t}^{B_f} = 1 - \mathbf{1}^\top \pi_{i,t}$.*

3 Interest Rates and Exchange Rate Risk Premia

The relationship between the equilibrium interest rates of the two countries allows us to assess the exchange rate risk premium implied by the model. While covered interest parity holds by no-arbitrage conditions in this complete market in equilibrium, uncovered interest parity (UIP) in generally violated due to a currency risk premium arising endogenously. Under certain conditions, interest rate differentials across *home* and *foreign* countries and the sign of the risk premium align to make a 'Carry Trade'—borrowing in the low-interest country, investing in the high-interest country and in expectation also gaining from currency movements—profitable. These investors would be earning the risk premium for taking on currency risk.

3.1 The Riskfree Interest Rates in Home and Foreign Country

Equilibrium interest rates are determined by investors' consumption and savings decisions, reflecting expectations of consumption growth rates and consumption volatility. Although there are two default-free securities available to investors that each provide a certain payoff, in terms of the numeraire good there is only one 'riskfree' asset: the *foreign* bond B_t^f, guaranteeing one future unit of the numeraire (*foreign*) good.[12] While the *home* bond also provides a risk-free return, this is

[12]Proposition 1 gives the results taking the *foreign* good Y_t^f to be the numeraire, though this is without loss of generality. Either of the two goods or a combination thereof can be used as numeraire good. In particular, the *relative* pricing

in the *home* consumption good. Neither of these is truly 'riskfree' in terms of anyone's chosen consumption basket (which is always comprised of both goods), but it is similar to the type of assets provided by the existing government bond markets. Either they pay in nominal currency, or inflation-protected assets such as TIPS, are based on an exogenously chosen basket that may or may not coincide with any particular investors consumption preferences.

To study uncovered interest parity, we compare the respective interest rates—as given in local currency (or real good)—and compare this to expectations on currency dynamics, or how investors expect these promised risk-free returns to be translated into the same currency.

Though it is perhaps most natural to express interest rates in terms of consumption growth and risk of the asset local to the interest rate, either interest rate can be rewritten in terms of consumption growth of the non-local good. The first expression may be easier to interpret for r_t^f, as it circumvents translation into the numeraire good.

$$r_t^f = \mu_{C_{Hf}} - \sigma^2_{C_{Hf}} f - \frac{\sigma_{C_{Hf}} \sigma_{\alpha_t}}{1 - \alpha_t^H} \tag{23}$$

$$= \mu_{C_{Hh}} + \mu_{p^h} - \sigma^2_{p^h} - \sigma^2_{C_{Hh}} - \mathsf{cov}_{C_{Hh},p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{\alpha,p^h} + \frac{1}{\alpha_t^H}\mathsf{cov}_{C_{Hh},\alpha} \tag{24}$$

Rewriting consumption growth in terms of economic fundamentals gives

$$r_t^f = s_f^F m_{Yf,t}^{(F)} + (1 - s_f^F) m_{Yf,t}^{(H)} - \sigma^2_{Yf,t}, \tag{25}$$

$$r_t^h = s_h^F m_{Yh,t}^{(F)} + (1 - s_h^F) m_{Yh,t}^{(H)} - \sigma^2_{Yh,t}, \tag{26}$$

where $s_f^F = \frac{\lambda_t \alpha^F}{1 - \alpha_t^H + \alpha^F \lambda_t}$ is F's consumption share of total consumption in *foreign* good Y_t^f, and $s_h^F = \frac{(1-\alpha^F)\lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ is F's consumption share of total consumption in *home* good Y_t^h, as given in (16). Both rates are positively related to aggregate consumption growth in their respectively local good, and negatively related to the associated aggregate consumption risk.

Under heterogeneity, rates are determined by the weighted average of investors' beliefs about

of assets remains identical. A consumption basket composed of fraction β of the *home* good Y_t^h and fraction $(1-\beta)$ of Y_t^f implies that relative goods prices are defined as $\beta p_t^h + (1-\beta) p_t^f = 1$.

aggregate consumption growth rate. Note that this average is higher than the expected growth rate under homogeneous beliefs. Taking expectations of equilibrium consumption processes shows $E^H[dC_{Hf,t}] + E^F[dC_{Ff,t}] = \mu_{Y_f} + s_f^F(1 - s_f^F)(\Delta m_t^{Y_h\,2} + \Delta m_t^{Y_f\,2})$. Both investors believe themselves to be making superior consumption and investment decisions compared to the other, and thus assume they will have higher future consumption growth than the other. This puts upwards pressure on r_t^f, which is offset by the disagreement risk the belief heterogeneity presents.[13]

On the surface, (25) indicates that a country's interest rate is driven exclusively by fundamentals in the local economy, seemingly refuting the importance of considering an open economy in the first place. However, there is an important indirect impact of cross-border expectations hidden in investors' consumption share. Both investors' beliefs about fundamentals in *both* economies are reflected in investors' state price densities and they feed into interest rate differentials through λ_t. This joint determination of interest rates based on overall views of the international economy is the source of the currency risk premium.

3.2 The Currency Risk Premium

In an economy with floating real exchange rates, the exchange rate is the multiplicative term that translates from one unit of measurement into the other, while no-arbitrage conditions are maintained. This is the idea behind the formulation of the '$M - M^*$' models of foreign exchange, which posit that discounting future cash flows using 'local' discount factor M_t is equivalent to discounting these cash flows using the foreign country's discount factor multiplied with today's exchange rate: $M_t^* S_t$. Exogenously determined consumption streams define M and M^*, giving exchange rates through the no-arbitrage condition. This technically casts the exchange rate as the relative value of one agent's consumption against consumption of the other agent; consumption markets are strictly segmented. For open economies, there needs to be a different measure that captures the difference between agents' relative consumption valuation and the exchange rate across countries.

From proposition 1, $\xi_t^H = \lambda_t \xi_t^F$ relates the two investors' state price densities to one another.

[13] These two effects exactly offset in the logarithmic utility environment studied in this paper.

If agents are fully homogeneous, this relationship (λ) would be constant. The corollary to models of the '$M - M^*$' setup is that if trade were allowed, exchange rates would be fixed. In this model, $\bar{p}_t$ captures the exchange rate, translates any one investor's state price density from denomination in terms of one country's good into the other: $\xi_t^h = \bar{p}_t \xi_t^f$.This relationship determines covered interest parity (CIP), and reflects the optimality of investors' consumption decisions: the marginal utility of consuming *foreign* goods is equal to the marginal utility of consuming *home* goods, scaled by the prevailing exchange rate, where both consumption and the exchange rate are determined in equilibrium.[14] This setup allows us to study whether the investor heterogeneity required to generate reasonable exchange rate dynamics as imposed in segmented markets can endogenously arise in unconstrained open economies.

The no-arbitrage condition $\xi_t^{Y_f} = \xi_t^{Y_h} / p_t^h$ gives the following relationship between interest rate differentials and exchange rate dynamics:

$$r_t^h = r_t^f - \mu_{p_h,t} + \sigma_{p_h,t} \kappa_t \tag{27}$$

where $\mu_{p_h,t}$ and $\sigma_{p_h,t}$ are the drift and diffusion of p_t^h, the expected appreciation of the *home* currency and its volatility.[15] The interest rate (in units of *home* 'currency') offered on *home* bonds, r_t^h, is equal to to the rate on *foreign* bonds r_t^f, less the expected appreciation of the *home* currency, plus the risk premium for taking on exchange rate risk.

Uncovered interest rate parity (UIP) posits that forward rates are an unbiased predictor of future spot rates, suggesting that currencies of countries with higher interest rates will depreciate over time, to compensate for the higher return investors can earn in the high-interest rate currency. (27) shows this implies either risk neutrality of investors ($\kappa_t = 0$), or exchange rates that are orthogonal to systematic risk ($\sigma_{p_h,t} \kappa_t = 0$). Then it would indeed be the case that $r_t^h > r_t^f$ only if $\mu_{p_h,t} < 0$, the

[14] In contrast, segmented markets necessarily define the exchange rate as the rate that equates marginal utiliy of consumption across investors, not goods. Goods are defined by who consumes them.

[15] Note that this is necessarily from the viewpoint of one investor, H or F. If they disagree on growth rates, they will likewise disagree on the market price of risk κ_t as well as the expected exchange rate appreciation $\mu_{p_h,t}$. Note that this arbitrage relationship will hold from either investor's view, with terms appropriately adjusted for the disagreement. Interest rates and diffusion are observable to both.

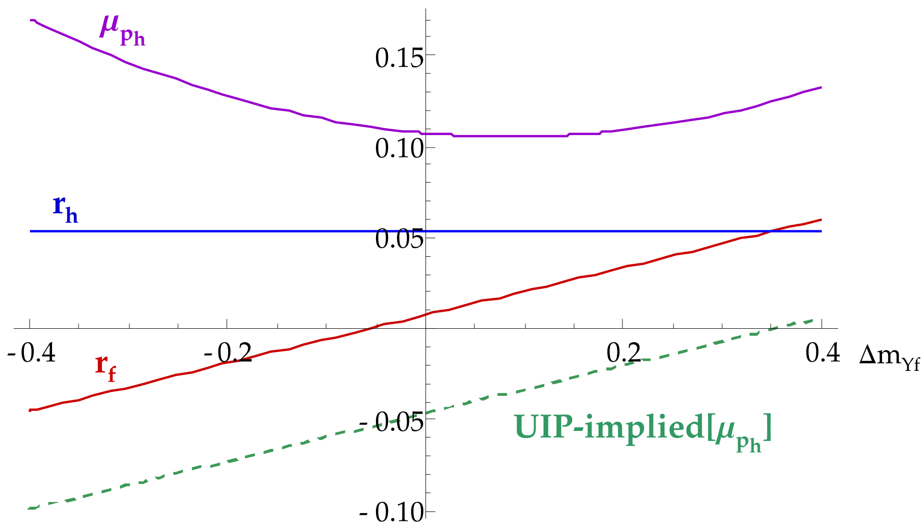


Figure 1: **UIP-Implied Depreciation** If UIP held, $r_t^f - r_t^h$ should be the expected appreciation of the *home* currency. This $\mu_{p_h,t}$ (dashed) implied by UIP theory is negative, in contrast to the true value (purple). The difference is the exchange rate risk premium.

higher interest rate currency is expected to depreciate. Empirical evidence shows that for a number of countries with consistent interest rate differentials, the opposite is indeed true.

Fig. (1) is an illustration of scenarios where a positive interest rate differential $r_t^h - r_t^f$ would, under UIP, suggest that a depreciation of the *home* currency is expected (dashed line), but the presence of the risk premium leads to a positive rate differential despite expected currency appreciation, $\mu_{p_h,t} > 0$. The remainder of this section discusses the risk premium in more detail.

The risk premium $\sigma_{p_h,t}\kappa_t$ is the exchange rate's covariance with the marginal utility of consumption; investors are compensated for carrying systematic risk. Only that component of exchange rate variation that is not orthogonal to priced risk matters, not total exchange rate variation. The empirical finding that exchange rate variation itself seems to matter is not inconsistent with (27) however. κ_t is the market price of risk in terms of the numeraire good. Rewriting this in terms of *home* goods prices gives

$$r_t^h = r_t^f - \mu_{p_h,t} + \sigma_{p_h,t}^2 + \sigma_{p_h,t}\kappa_t^{home}, \tag{28}$$

separating components of the 'world' market price of risk into local production risk and exchange rate risk. Here this decomposition is straightforward as the *foreign* good has been determined the reference price. The same decomposition into local and international risk components would hold no matter the numeraire good.

Structurally, interest rates in this two-country/two-good world are similar to the rate in a single-country economy with only one good: high output growth rates imply high interest rates, high aggregate risk in the form of volatile output lowers interest rates.When total consumption demand is split across two different goods, what becomes relevant for determining market prices of risk and interest rates is the growth of output *relative* to the growth of demand for that good. The equilibrium state price density is the value of consumption, and can be expressed in terms of either good: $\xi_t^H = \frac{1-\alpha_t^H}{C_H^f \psi_H} = \frac{\alpha_t^H}{C_H^h \psi_H \bar{p}_t}$ depends on both demand and supply for the respective goods.[16] While output is an exogenous process in a Lucas Tree economy, total demand depends on (time-varying) preferences α_t^H and α^F as well as the respective weight λ_t that pins down consumption share. Hence, it is not simply countries' output growth and its associated production risk that determines interest rates, but the growth of output relative to growth in demand, and how demand covaries with output, that denotes aggregate risk in this environment. Unsegmented goods markets are critical for this. If all local output is always consumed by local agents, demand is necessarily given by one agent, as in a benchmark representative environment. This would render $\bar{p}_t$ and λ_t, inseperable.

Defining aggregate demand as $\mathcal{D}_{Y_h} = \alpha_t^H + (1-\alpha^F)\lambda_t$ for the *home* good and $\mathcal{D}_{Y_f} = 1-\alpha_t^H + \alpha^F \lambda_t$ for the *foreign* good, allows us to rewrite (27) using $\xi_{h,t} = \mathcal{D}_{Y_h,t}/Y_t^h$ and $\xi_{f,t} = \mathcal{D}_{Y_f,t}/Y_t^f$ as

$$r_t^h = r_t^f - \mu_{p_h,t} - \sigma_{\xi_f,t}\sigma_{p_h,t}, \tag{29}$$

$$r_t^h = r_t^f - \left(\mu_{\mathcal{D}_h/Y_h} - \mu_{\mathcal{D}_f/Y_f} + \sigma^2_{\mathcal{D}_f/Y_f} - \sigma_{\mathcal{D}_h/Y_h}\sigma_{\mathcal{D}_f/Y_f}\right) - \sigma_{\mathcal{D}_f/Y_f}\left(\sigma_{\mathcal{D}_h/Y_h} - \sigma_{\mathcal{D}_f/Y_f}\right). \tag{30}$$

There are technically two scenarios in which the carry trade is profitable: if $r_t^h > r_t^f$, the risk premium in (27) would have to be positive, if $r_t^h < r_t^f$, it would have to be negative. Since the economy

[16] Naturally, the same can be expressed from the point of view of investor F, with appropriate changes through λ_t.

is set up symmetrically, the intuition for the conditions is fundamentally the same in both cases, so only the first of these two cases is discussed here.

When $r_t^h > r_t^f$, UIP predicts $\mu_{\bar{p}_t} < 0$. A violation of UIP occurs when instead $\mu_{\bar{p}_t} > 0$, which implies $\sigma_{\xi_f,t}\sigma_{p_h,t} < 0$. The latter is the risk premium: a negative covariance between $\bar{p}_t$ and state price density ξ_t means the *home* currency is valuable in good times, making it a bad hedge against systematic risk; its return must be higher to compensate, $\mu_{\bar{p}_t} > 0$.

For r_t^h to be higher than its *foreign* counterpart r_t^f, the expected growth rate of '*home* demand relative to supply' must be lower than for the *foreign* country: the more investors expect a country's output growth to be able to cover demand growth, the higher that country's interest rate is.[17] The risk premium is determined by the risk of satisfying future demand with future supply.

$$\text{If } \sigma_{\xi_t}\sigma_{\bar{p}_t} = \kappa_{f,t}\left(\kappa_{h,t} - \kappa_{f,t}\right) < 0 \text{ when } r_t^h > r_t^f \text{ , carry trade is profitable.} \tag{31}$$

Setting the *foreign* good to be the numeraire, $\kappa_{f,t}$ can be seen as the 'world' market price of aggregate risk that prices all financial assets. Reasonable preference parameters must be such that this is positive, which means the differential in brackets in (31) must be negative: the volatility of demand relative to supply must be more volatile for *home* country's good than *foreign* country's.

While interest rates reflect expectations about output growth keeping up with demand growth for the *home* good, the *home* currency is expected to appreciate—giving rise to the carry trade—if demand for the country's good is highly volatile *relative* to output. This is the actual source of risk in a multiple good economy: if demand were perfectly correlated with output, preferences (or whatever makes demand shift) would be a natural hedge for economic production risk, reducing or eliminating risk premia. In contrast, imperfectly correlated time variation in demand can also exacerbate risk premia. From the definition of $\mathcal{D}_h/Y_h$ and $\mathcal{D}_f/Y_f$ the ratio of demand to supply is more likely to be volatile if taste shocks are volatile ($\sigma_{t,\alpha}$ is high) or the *foreign* investor is rich (λ_t is high) and carries disproportionate amounts of aggregate risk ($\Delta\vec{m}_t$ is positive).[18]

[17] To ensure positive interest rates in a country, supply must be expected to grow faster than demand. The reverse can lead to negative rates.

[18] For a profitable carry trade when $r_t^f > r_t^h$ the intuition is analogous: the ratio of demand relative to supply for the

The model suggests that one should empirically find a carry trade generate high returns where the high interest rate country has a growing economy on the production side that can, in expectation, keep up with future demand growth, but where demand is erratic relative to supply, because the demand is dependent on exports to a (rich) country whose consumption (and thus also demand for the non-local good) loads heavily on aggregate consumption risk. This is consistent with Jylha, Suominen, and Lyytinen (2008) and Ranaldo and Soderlind (2010), who find that carry returns are positively correlated with the risk premium on equity.

In terms of economic fundamentals and endogenous state variable λ_t the risk premium $\kappa_t^f \sigma_{\bar{p}_t}$ is

$$\kappa_t^f = \left[-s_f^F \Delta m_t^h \ , \ \sigma_{Y_f} - s_f^F \Delta m_t^f \ , \ \frac{1}{(1-\alpha_t^H + \lambda_t \alpha^F)} \sigma_{\alpha,t}\right] \tag{32}$$

$$\sigma_{\bar{p}_t} = \left[-\frac{\lambda_t(\alpha^F - (1-\alpha_t^H))}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \Delta m_t^h - \sigma_{Y_h}, -\frac{\lambda_t(\alpha^F - (1-\alpha_t^H))}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \Delta m_t^f + \sigma_{Y_f}, \frac{1+\lambda_t}{\mathcal{D}_{Y_h}\mathcal{D}_{Y_f}} \sigma_{\alpha,t}\right] \tag{33}$$

where $\mathcal{D}_{Y_h}$ and $\mathcal{D}_{Y_f}$ is demand as defined above.

Fig. (2) illustrates that this risk premium is indeed positive for a wide range of reasonable parameters that can be considered reasonable. The parameters used for this illustration are as depicted in the table below, although graphs look similar for a variety of parameters.[19]

The four plots show how the interest rate differential $r_t^h - r_t^f$ and the risk premium $\kappa_t \sigma_{\bar{p}}$ vary across λ_t, for different degrees of belief heterogeneity Δm_t^h and Δm_t^f. The intuition for the impact of investor heterogeneity on financial market equilibrium is most easily seen through preferences. Due to the home bias in consumption, any additional unit of wealth an agent attains will partially be consumed. Of that additional consumption, a disproportionate amount will go to consumption of the local good—pushing up the price of that good relative to the non-local good. Stock prices are the expected discounted future value of output: for good $i = H, F$ that is $p_t^i Y_t^i$. The higher the price of the good, the higher the value of its output, the higher the associated stock price. It is through this channel that investor heterogeneity affects exchange rates: Δm_t captures differences in investors

foreign good is expected to grow more slowly, while this ratio being more volatile than the demand-supply ratio of *home* makes the *foreign* currency appreciate in expectation.

[19]Note that $r_t^h - r_t^f > 0$ requires a higher expected growth rate for the *home* country.

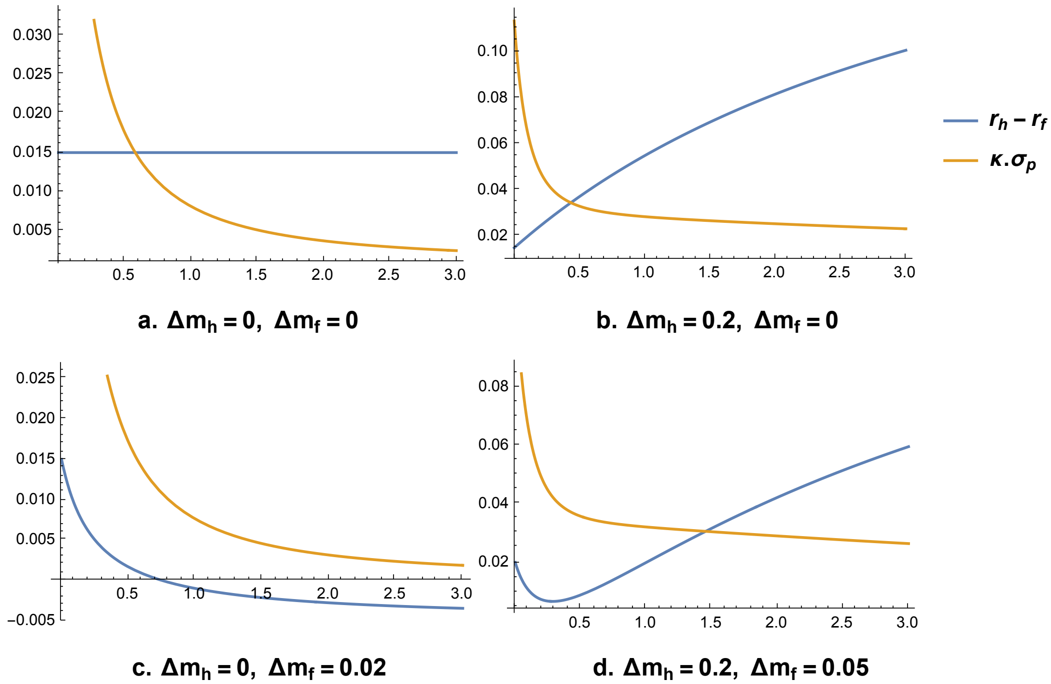


Figure 2: **Interest rate differentials and risk premium** Plots a. to d. show how the interest rate differential and currency risk premium vary across λ_t, the relative weight of the *foreign* country in the economy, for different levels of disagreement. (Note that for high levels of Δm_t^f the interest rate differential would become negative)

Description	Parameter	values
expected growth rate in foreign output	m_f^H	0.015
expected growth rate in home output	m_h^H	0.03
volatility of foreign output	σ_{Y_f}	0.03
volatility of foreign output	σ_{Y_h}	0.03
volatility of demand shock	σ_α	0.06
domestic bias in consumption of agent H	α_t^H	0.8
domestic bias in consumption of agent F	α^F	0.8

Table 1: Model parameters used for fig. (2). Graphs are representative of a broad range of parameters.

portfolio choices: the more optimistic investor takes on more aggregate risk by investing more in the relevant stock. Who owns more becomes relevant for the feedback effect: an agent whose positive shock to wealth comes from high ownership of his own local stock (home bias in portfolio choice), will impart a positive feedback effect into stock prices by further increasing the price of his local good via consumption choices. If his positive shock to wealth comes from returns in the non-local stock, he will channel part of that benefit back into his own country through consumption. this opposite impact on exchange rates depending on *who* owns risky assets is the reason that the magnitude, but also the sign of Δm_t matters.

Rewriting eq. (27) to be more reminiscent of the classic forward premium regression $E[dp_t] = \alpha + \beta[r_t^f - r_t^h] + \epsilon_t$ shows that the risk premium may lead to an omitted variable bias in the regression.

$$\mu_{p_h,t} = r_t^f - r_t^h + \sigma_{p_h,t}\kappa_t$$

Consider eqs. (25) and (26), which show that consumption shares s_h^F and s_f^F as well as expected ouptut growth determine interest rates. Likewise, the decomposition of the risk premium indicates that the same state variables drive $\sigma_{p_h,t}\kappa_t$. How this risk premium covaries with the interest rate differential determines the bias in the UIP regression.

3.3 Negative Skewness and Funding Restrictions

A number of papers have brought up the importance of funding restrictions that instituational investors are subject to as a reason for negative skewness in currency returns, e.g. Jylha, Suominen, and Lyytinen (2008).[20]he Gaussian structure of returns in this model cannot support a skewness premium as reported in the empirical literature. However, under some conditions the correlation between $\bar{p}_t$ and its own variance—which is itself time-varying—is negative. With time-varying volatility, implied skewness extracted from option prices could potentially be due to an implied (i.e. anticipated) negative correlation between the (high-interest) currency and it's own volatility

[20]T

between the trading date and the maturity date of the option.[21] To coincide with what would appear to be a skewness premium, the covariation of expected currency returns and currency volatility must be negative. To wit, for the scenario $r_t^h > r_t^f$ discussed here, skewness would be detrimental to the investor that sets up a carry trade position only if exchange rate volatility rises as the exchange rate itself falls. This type of 'skewness' would appear in a time series when the covariance between $\bar{p}_t$ and its own volatility $\sigma_{\bar{p}}$ is negative. Using (33) we can show that this covariance is negative if aggregate risk is unevenly distributed across the two agents. In particular, if the *foreign* investor carries sufficiently high amounts of aggregate risk, in which case the carry trade is profitable in the above scenario.

Alternatively, one can think of this form of artificially generated skewness in the time series as the joint behavior of expected apreciation and exchange rate volatility. Fig. (3) plots expected (instantaneous) appreciation of the *home* currency and the variance of the exchange rate (alongside the risk premium itself). The plots are for varying degrees of investor heterogeneity, as these parameters are a) the most likely to change significantly over shorter time horizons, and b) the skewness premium has been linked to large movements in investor portfolio holdings (such as e.g. hedge funds); in this model belief heterogeneity is the parameter most closely related to this, as it directly impacts portfolio holdings and changes in Δm_t would cause large trades to occur.

Most interesting about the plots is perhaps that the parameter space where a change in investor disagreement leads to expected appreciation and volatility heading in opposite directions is indeed *not* found at extreme levels of disagreement. It is indeed most prominent in plot b.: when F is only slightly more optimistic about output growth in *foreign* ($\Delta m_t^f = 0.1$) and he moves to be slightly more optimistic about *home* country as well (moving slightly to the right of the origin, where Δm_t^h is small but positive), in this scenario will volatility of exchange rates rise, while the expected appreciation of *home* falls. The relationship between return and volatility is most sensitive when investors are close to agreement.

The skewness premium has been linked to the funding constraints experienced by institutional

[21]Though it would be difficult to assess how volatile this relation between returns and volatility is in the model and whether it would be sufficient to generate the observed 'skewness premium'.

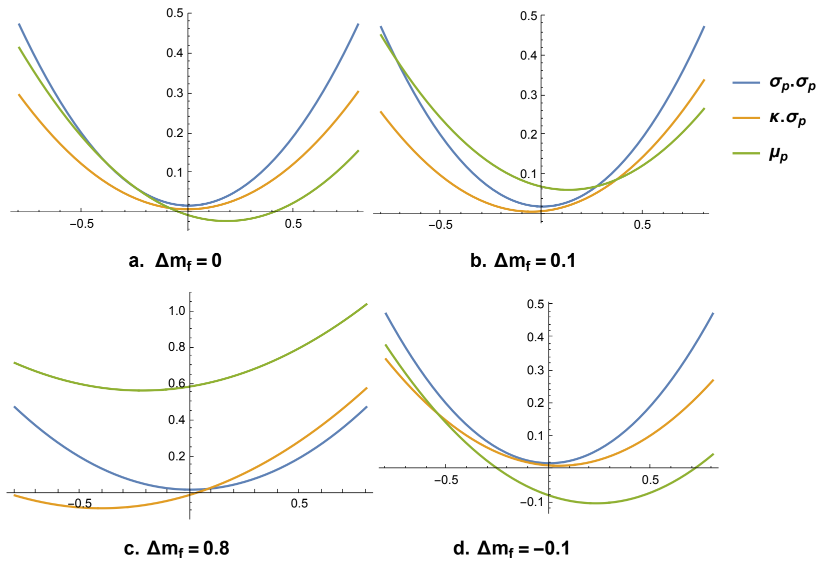


Figure 3: **Expected depreciation and volatility** Plots a. to d. show how the expected appreciation of the *home* currency and $\mu_{\bar{p}}$ and volatility $\sigma_{\bar{p}}$ move with disagreement Δm_t^h.

investors, the group most likely to have large positions aiming to benefit from carry between currencies. The idea behind this is currency crash risk generated by a market friction. Investors experiencing sudden liquidity constraints have to unwind their positions quickly. Pulling money out of the high-interest country (the long leg of the trade), that country's currency falls, thus undoing any profit of the carry trade.

I impose a funding constraint in the form of an exogenously imposed limit on the leverage taken up by one of the investors, the *foreign* investor F. Such constraints are often the result of the difficulties in contingent contracting: bankruptcy costs and agency costs lead to limitations on leverage for most investors, either explicitly or implicitly through, e.g. margin constraints. In the aftermath of the 2007-2008 financial crisis and its repercussions around the world, the debate about leverage restrictions—and whether it must be in- or decreased—was reignited. Most investors engaged in the carry trade are institutional investors or money managers, who often face such constraints.Although the constraint analyzed here is exogenous, comparative statics on the constraint parameter can give some insight into how markets would react to tightening or loosening the level of leverage restrictions.

Utility functions and budget constraints remain as given in (2), (3) and (13).

While H is free to optimize his investment, F is limited in the amount of leverage he can take on by borrowing in bond markets: his positions in stocks cannot exceed a proportion $\eta > 1$ of his total wealth. The constraint can be expressed as

$$\mathtt{I}^{\top}\pi_{F,t} \leqslant \eta \, ; \qquad \mathtt{I} = [1,1,0]^{\top} \tag{34}$$

where $\pi_{F,t} = [\pi_{F,t}^{S_h}, \pi_{F,t}^{S_f}, \pi_{F,t}^{B_h}]^{\top}$ is the vector of F's portfolio holdings of investor in both stocks and the *home* bond.[22]

When the imposed constraint binds, optimal risk sharing is hindered by limiting the investor most willing to take on the risk from providing liquidity to the market. Prices in all financial markets have to accommodate this, and incentivize the other, unconstrained investor, to supply this missing liquidity. In order for such a constraint to lead to a significant rebalancing of portfolios, it must be that it is suddenly and unexpectedly imposed on the market.

The assumption of differences in beliefs is a technically tractable way to allow for constraints to bind with different degrees of severity—for a given level of leverage limitation η. How strict the constraint is, and how severely investors find themselves constrained by it, are two notions of a constraint's severity, but have a different impact on equilibrium. How the constrained investor adjusts his portfolio to compensate for the imposed restriction will depend on his beliefs about the alternative investment opportunities. If the binding limit on leverage η remains stable the same but the constrained investor's beliefs change, equilibrium market rates will change, despite the fact that overall leverage of the investor cannot change.

Constraints distort the desired portfolio choice: constrained investors seek alternative assets, in a manner that allows them to replicate their desired portfolio as closely as possible. The presence of constraints prevents investors from trading optimally, and the constrained investor must

[22]Satisfaction of the budget constraint implies $\pi_{F,t}^{B_f} = 1 - \mathbf{1}^{\top}\pi_{F,t}$.

choose alternative investments that are permissible. How the optimal portfolio adjustments are determined, and how this distortion affects equilibrium, can be seen in the state price density ξ_t^F of investor F. This form allows us to distinguish the constrained investor's true assessment of investment opportunities from the density that is reflected by his actual portfolio choices. The distortions created in the equilibrium state price density by the constrained portfolio choices are captured by two parameters.[23] When the constraint binds, F's state price density changes from (15) to

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t)\right)\xi_t^F dt - \vec{\kappa}_{\upsilon,t}^{F^\top}\,\xi_t^F d\vec{W}_t^{(F)}. \tag{35}$$

where $\vec{\kappa}_{\upsilon,t}^F$ reflects F's true beliefs about the risk-return tradeoff as well as the restrictions the constraint places on his portfolio:

$$\vec{\kappa}_{\upsilon,t}^F = \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) + \vec{\sigma}_{S,t}^{-1}\upsilon_t \mathtt{I}. \tag{36}$$

υ_t and $\delta(\upsilon_t)$ are scalar parameters that capture the effect of the leverage constraint on investor F's investment decisions: the constraint changes the relative attractiveness of all assets, including both bond markets, which both serve as a source of leverage for the investor.

Investors' beliefs about investment opportunities determine whether the constraint will bind for F: if he is sufficiently optimistic relative to investor H about economic growth rates in at least one of the two countries, the investment restriction will pose a problem. In order for markets to clear, prices must adjust. Fig. (4) shows the conditions under which the two constraints will, respectively, bind.

When the constraint binds, investor F's portfolio holdings appear inconsistent with the publicly observable riskfree interest rate. In contrast to the unconstrained investor H, his portfolio is distorted: he must reallocate the funds that he would like to invest into the stock markets to the bond markets or immediate consumption, making it appear as though he is making decisions based on different economic parameters, an adjusted interest rate $(r_t + \delta(\upsilon_t))$. The following proposition

[23]Cvitanic and Karatzas (1992) introduced the methodology to incorporate investment constraints on portfolio choice. Other related papers are e.g. He and Pearson (1991) and Cuoco (1997).

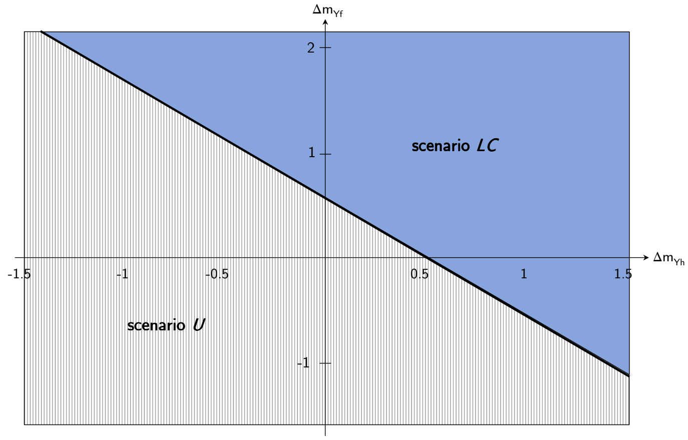


Figure 4: **Binding Leverage Constraint** The constraint binds ('scenario LC') if F is sufficiently optimistic about at least one of the two countries' investment opportunities. Strong optimism vis-a-vis one country is sufficient, as e.g. in the case of a 'home bias', in the second quadrant of the graph: $\Delta m_t^{Y_f} > 0$ and $\Delta m_t^{Y_h} < 0$.

details the constrained equilibrium.

Proposition 2. *Still taking good Y_t^f to be the numeraire, equilibrium stock and bond prices retain the form detailed in* (20).

$\bar{p}_t = p_t^h/p_t^f = \xi_t^h/\xi_t^f$ *holds, taking into consideration the changed dynamics of ξ_t^j , $j = h, f$. In particular,*

$$d\lambda_t = d\left(\frac{\psi_H \xi_t^H}{\psi_F \xi_t^F}\right) = \lambda_t \Delta\vec{\kappa}_t^\top d\vec{W}_t^{(H)} \tag{37}$$

where $\Delta\vec{\kappa}_t^\top = \left[\Delta\kappa_t^h, \Delta\kappa_t^f, \Delta\kappa_t^\alpha\right]$ capture differences in investors' market prices of home, foreign, and demand risk, which depends on the binding of the constraint: $\Delta\vec{\kappa}_t = \Delta\vec{m}_t^Y + \vec{\sigma}_{S,t}^{-1}(\upsilon_t \mathtt{I})$. Adjustment term υ_t is non-positive iff the constraint is binding, and zero otherwise. Portfolio weights of investors H and F are, respectively,

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t \mathbf{1}\right), \tag{38}$$

$$\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1} \upsilon_t \mathtt{I} & \text{if } \upsilon_t < 0 \\ (\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t \mathbf{1}\right) & \text{otherwise} \end{cases} \tag{39}$$

where $\upsilon_t = \min\left(\frac{\eta - \mathtt{I}^\top(\vec{\sigma}_{S,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right)}{\mathtt{I}^\top(\vec{\sigma}_{,t}^{-1})^\top \vec{\sigma}_{S,t}^{-1}\mathtt{I}}, 0\right)$*:*

$$\upsilon_t = \begin{cases} \frac{-\sigma_{Yh}\sigma_{Yf}\sigma_\alpha^2\left[\Delta m_t^{Y_h}\sigma_{Yf} + \Delta m_t^{Y_f}\sigma_{Yh} - (\eta-1)(1+\lambda)\sigma_{Yh}\sigma_{Yf}\right]}{(\eta-1)^2\lambda_t^2(\alpha_t^H + \alpha^F - 1)^2\sigma_{Yh}^2\sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2)\sigma_\alpha^2} & \textit{if } \Delta m_t^{Y_h}\sigma_{Y_f} + \Delta m_t^{Y_f}\sigma_{Y_h} > (\eta - 1)(1 + \lambda_t)\sigma_{Y_f}\sigma_{Y_h}, \\ 0 & \textit{otherwise}. \end{cases} \tag{40}$$

The collateral adjustment is $\delta(\upsilon_t) = -\eta\upsilon_t$*.*

$(\vec{\sigma}_{S,t}^{-1})^\top(\vec{\sigma}_{S,t}^{-1}\upsilon_t^{case}\mathtt{I}_{case})$ is the adjustment to F's portfolio in response to the binding leverage constraint. Having to reallocate his investment, F seeks assets—or portfolios thereof—that are highly correlated with the desired, but inaccessible investment. Thus, assets' covariance structure plays a key for the reallocation. The adjustment term υ_t captures the wedge that a binding constraint drives between F's true expectations about fundamental growth rates and the expectations reflected in asset prices via portfolio choice. The magnitude of this distortion to F's state price density depends on two characteristics: how strict the constraint is, i.e. the level of η, as well as how severely investor F is constrained, ie. the 'distance' between his desired position and the permissible one, conditional on a given η. The latter is determined by expectations about investment opportunities, $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$.

When F is optimistic regarding at least one or both of the countries' growth rates, his leverage constraint binds, and his true expectations are not accurately reflected in his investment choices. Rewriting r_t^f from (25) in terms of investors' disagreement using (10) and (36) shows how the leverage constraint distorts the link between interest rates and expected consumption growth rates.[24]

$$\begin{aligned} r^f_{t,U} &= s^F_f m^{(F)}_{Yf,t} + (1-s^F_f) m^{(H)}_{Yf,t} - \sigma^2_{Yf,t} \\ &= m^{(H)}_{Yf,t} - \sigma^2_{Yf} + s^F_f \sigma_{Yf} \Delta m^{Y_f}_t \end{aligned} \quad (41)$$

$$\begin{aligned} r^f_{t,LC} &= m^{(H)}_{Yf,t} - \sigma^2_{Yf} + s^F_f \sigma_{Yf} \Delta \kappa^f_t \\ &= r^U_t + s^F_f \sigma_{Yf} (\vec{\sigma}^{-1}_{S,t} \upsilon_t \mathtt{I})_{\text{el.2}}, \end{aligned} \quad (42)$$

where $(\cdot)_{\text{el.2}}$ denotes the 2nd element of the vector $(\cdot)$. (42) shows that when the constraint binds, the induced reallocation of F's portfolio puts downward pressure on interest rates. $(\vec{\sigma}^{-1}_{S,t} \upsilon_t)_{\text{el.2}}$, reflects the wedge that the constraint drives between F's true disagreement about consumption growth rates and those reflected in his portfolio. This term is negative whenever the constraint binds, F holds fewer risky assets than he would optimally like to. There are two ways to see the intuition: as F is constrained from showing his true (optimistic) beliefs in his portfolio, the expected consumption growth rates that are implied by $r^f_{t,LC}$ are lower than true expected growth rates. Alternatively, one can consider the portfolio side: F's leverage under the binding constraint is lower than it would be absent the constraint, under $r^f_{t,U}$. Lower effective demand for leverage reduces its price.

The magnitude of this distortion depends on three elements. First, the level of η. The amount of leverage F is permitted to take captures the 'strictness' of the constraint. Second, disagreement parameters $\Delta m^{Y_h}_t$ and $\Delta m^{Y_f}_t$. These determine how much of the economy's risk F would optimally carry in an unrestricted market, and therefore capture how 'severely' the constraint affects F. Third, the covariance structure of asset markets, $\vec{\sigma}_{S,t}$. Covariance determines how F will optimally use the other available assets to construct the best possible substitute portfolio that minimizes the impact of the constraint. This interaction creates a correlation between interest rates and stock market volatilities beyond the contribution of fundamental risk to interest rates.

[24] The change to r^h_t follows analogously.

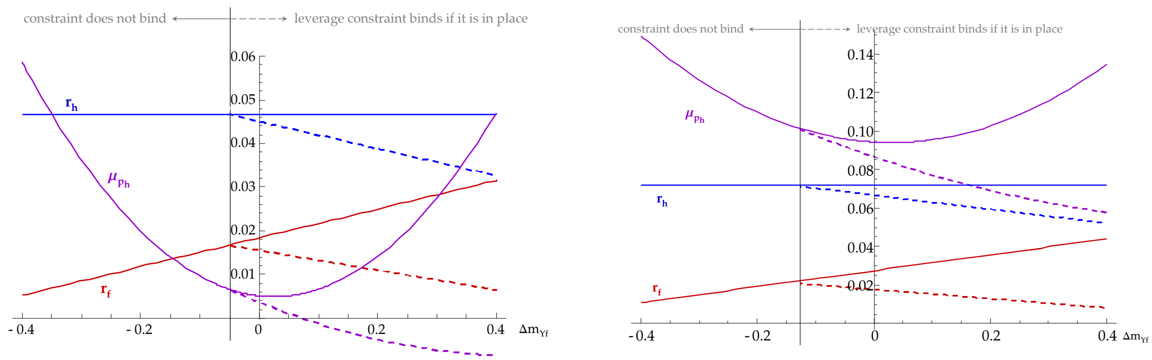
While $r^f_{t,U}$ depends positively on $\Delta m_t^{Y_f}$, it is independent of $\Delta m_t^{Y_h}$. This is not the case in the constrained equilibrium.

$$
\begin{aligned}
\frac{\partial r^f_{t,LC}}{\partial \Delta m_t^{Y_h}} &= && s_f^F \sigma_{Yf} \frac{\partial(\vec{\sigma}_{S,t}^{-1} \upsilon_t \mathtt{I})_{\text{el.2}}}{\partial \Delta m_t^{Y_h}} = \frac{-(1+\eta) s_f^F \sigma_{Yh} \sigma_{Yf}^2 \sigma_\alpha^2}{\left((\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2) \sigma_\alpha^2\right)} < 0, \\
\frac{\partial r^f_{t,LC}}{\partial \Delta m_t^{Y_f}} &= && \frac{\partial r^f_{t,U}}{\partial \Delta m_t^{Y_f}} + s_f^F \sigma_{Yf} \frac{\partial(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{LC})_{\text{el.2}}}{\partial \Delta m_t^{Y_f}} = s_f^F \sigma_{Yf} \left(1 - \frac{(1+\eta) \sigma_{Yh}^2 \sigma_\alpha^2}{(\eta-1)^2 \lambda_t^2 (\alpha_t^H + \alpha^F - 1)^2 \sigma_{Yh}^2 \sigma_{Yf}^2 + (\sigma_{Yh}^2 + \sigma_{Yf}^2) \sigma_\alpha^2}\right) > 0.
\end{aligned}
$$

As differences in beliefs $\Delta m_t^{Y_h}$ or $\Delta m_t^{Y_f}$ rise, F would optimally take on more stock market risk in this economy than he is able to. This unrealized demand for borrowing pushes down interest rates. Because F is constrained in his *joint* stock holdings, a sudden change in optimism about either country can trigger this effect. The constraint transmits effects of investor heterogeneity about one country's fundamentals to other countries' interest rates. In an economy with investment frictions, interest rates are more sensitive to other countries' investment opportunities than would be implied by frictionless models.

Changing the stringency of the imposed constraint η —essentially regulatory action—has the predictable impact. $\partial r_t^{LC}/\partial \eta > 0$ whenever the constraint binds. Lowering η implies a change in regulation that forces F to liquidate part of his portfolio to lower his leverage. Intuitively, as known from models with a single 'world' riskfree bond, this leads to a higher demand for the risk-free bond, markets clear at a lower interest rate r_t^f.

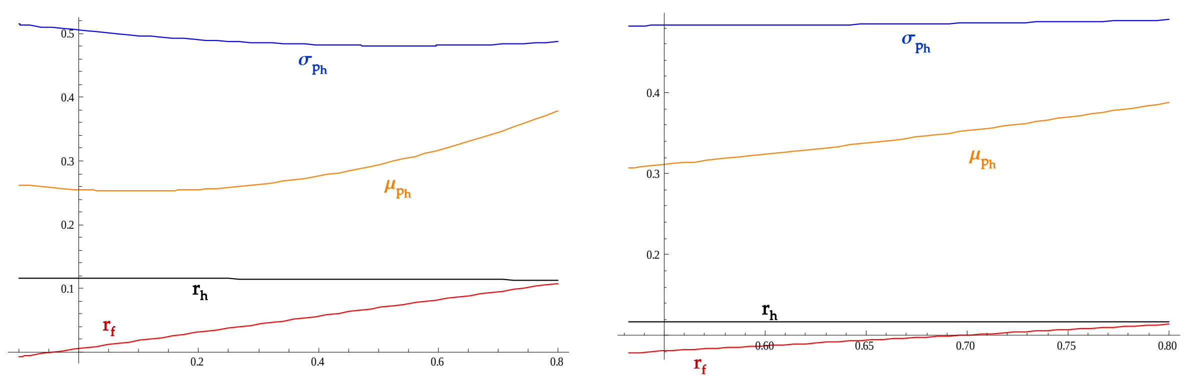
Fig.(5) illustrates the effect of a suddenly imposed leverage constraint on interest and exchange rates. Comparing the dashed lines in graphs (a) and (b) indicates the effect of suddenly imposing a leverage constraint on a previously unconstrained economy. It shows r_t^h, r_t^f and expected appreciation of the *home* curreny, both for the unconstrained case U (solid), as well as for the leverage-constrained case LC (dashed).The immediately binding leverage constraint would lead to the sudden drop in exchange rate expectations μ_{p_h}, moving against carry traders. Brunnermeier, Nagel, and Pedersen (2008) show that currency pairs that exhibit profitable carry trade opportunities tend to have a negatively skewed distribution. This is interpreted as 'currency crash risk': when a currency depreciates, volatility is higher. Although distributions here are not skewed, this



(a) $\Delta m_t^{Y_h} = 0.2$ (small) (b) $\Delta m_t^{Y_h} = 0.5$ (large)

Figure 5: **UIP Violation—Unconstrained and Constrained** *home* and *foreign* interest rates r_t^h and r_t^f as well as expected appreciation of the *home* currency p_t^h, in the unconstrained (solid) and constrained case (dashed).

downward adjustment of expected returns has a similar effect if such an event were part of a data sample.



(a) $\eta = 1.1$ (strict leverage constraint) (b) $\eta = 2.5$ (lenient leverage constraint)

Figure 6: **Negative 'Skew' Due to Funding Constraint** Only when constrained investor F is optimistic about investment opportunities in the high-interest-rate country will the exchange rate p_t^h be negatively skewed: As $\Delta m_t^{Y_f}$ falls, exchange rate volatility σ_{p_h} rises and expected appreciation μ_{p_h} falls.

Although the above shows that suddenly binding constraints can indeed affect currency markets in a way that poses a risk for investors with carry trade positions, comparing panels (a) and (b) in fig. (6) shows that negative skewness of the high-interest rate currency arises only when a very strict constraint is in place, and the effect is relatively weak. As investors' assessment of *foreign* country's investment opportunities changes, expected appreciation and volatility move in opposite directions. Both graphs are plotted for $\Delta m_t^{Y_h} = 0.8$, indicating that the *foreign* investor expects high growth rates in the *home* country.

Indeed, negative skewness only arises for high levels of $\Delta m_t^{Y_h}$; high interest rates can only be sustained by high expectations about economic growth. In panel (a), volatility rises and expected appreciation falls—the left tail of the negative skew—as $\Delta m_t^{Y_f}$ falls. The fairly equal sharing of *foreign* market risk while *home* risk becomes more unevenly shared produces a lopsided exposure, which is reflected in exchange rates.

This effect occurs when the leverage constraint η is low, indicating a strict constraint. The stricter the constraint is, the more F's portfolio is distorted for given beliefs about growth rates. The magnitude of this distortion creates the negative skew. Volatility, as the square root of variance, is affected by the absolute magnitude of investor heterogeneity $\Delta m_t^{Y_h}$ and $\Delta m_t^{Y_f}$, whereas beliefs about the two countries growth rates will have opposite directional effects on expected exchange rate movements μ_{p_h}.

This, albeit tentative, result suggests that negative skewness found in currency options is not necessarily the result of concern about a crash in economic fundamentals of high-interest rate countries, but could also be an indicator of anticipated fluctuations in sentiment and disparity of international risk sharing.

4 Conclusion

The paper studies a two-country open economy model that endogenously generates currency risk premia.

The model shows that Uncovered Interest Parity will be violated under certain conditions, giving rise to the 'carry trade'—the profits from investing in a high-interest-rate bond with money borrowed in a low-interest-rate country will not be offset by a commensurate depreciation of the high-interest currency.

Interest rate differentials reflect investors beliefs about the ability of the countries' respective output to keep up with the growth in demand. However, due to integrated goods as well as financial markets, demand for goods is sensitive to endogenous changes in the wealth distribution across investors. If demand for a country's good relative to its supply is risky, its currency is a bad hedge against systematic risk, its return must be high, the currency appreciates on average. These conditions, under which UIP is violated, are more likely to occur when the investor foreign to this country is rich and carries more aggregate risk.

This suggests that one should find profitable carry trade in situations where the high interest rate country is a growing economy that can, in expectation, keep up with future demand growth, but where demand for its good is erratic due to being dependent on exports to a country whose wealth is sensitive to stock market risk.

The exchange rate risk premium is simply compensation for systematic risk in a Gaussian economy, but the model is consistent with finding skewness in the time series of exchange rates. The parameters determining exchange rates are endogenous, and thus time-varying. For the carry trade skewness is detrimental if the volatility of the exchange rate rises just as the exchange rate moves against the carry trade. This covariance is negative, creating the impression of a negative skew in the data, if aggregate risk is unevenly distributed across the two agents. Thus, the same condition that makes a UIP violation (i.e. a profitable carry trade) between two countries more likely, also generates exchange rate dynamics that are consistent with finding a skewed distribution in the data.

This paper studies exchange rate dynamics in open economy with unsegmented goods as well financial markets in order to better understand currency premia and the conditions under which carry trades are profitable. Many currency pairs where this is the case have reasonably integrated markets, therefore separating out the effects of segmentation to provide hypotheses on these markets is valuable. Export dependency of high-growth countries, as well as the allocation of aggregate risk across investors is shown to have a significant impact on currency risk premia.

Appendix

A Alternative Constraint: Limit on non-domestic stockholding

conceptually similar results arise in the presence of a different type of constraint: a limit on F's investment into the stock market abroad, S_t^h. Where the leverage constraint allows the investor the freedom to optimize the allocation among stocks even if not the total amount of stock holdings, this type of constraint is more one-sided. This makes it more difficult to construct an optimal portfolio of stocks, but conversely leaves more freedom in other, unconstrained assets, to compensate for the constraint. Referring to this constraint as ND—a restriction on non-domestic stocks—the restriction is formalized as follows.

$$\text{non-domestic stockholding constraint:} \qquad \mathtt{I}_{ND}^{\top}\pi_{F,t} \leqslant \varphi \tag{43}$$

where $\mathtt{I}_{ND} = [1, 0, 0]^{\top}$, and $\pi_{i,t} = [\pi_{i,t}^{S_h}, \pi_{i,t}^{S_f}, \pi_{i,t}^{B_h}]^{\top}$ is the vector of portfolio holdings of investor i in both stocks and the *home* bond.

case ND: equilibrium when investor F faces a constraint on holdings of S_t^h

$$\pi_{Ht} = (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(H)} - r_t\mathbf{1}\right), \tag{44}$$

$$\pi_{Ft} = \begin{cases} (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) + (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\upsilon_t^{ND}\mathtt{I}_{ND} & \text{if } \upsilon_t^{ND} < 0 \\ (\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right) & \text{otherwise} \end{cases} \tag{45}$$

where $\upsilon_t^{ND} = \min\left(\frac{\varphi - \mathtt{I}_{ND}^{\top}(\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\left(\vec{m}_{S,t}^{(F)} - r_t\mathbf{1}\right)}{\mathtt{I}_{ND}^{\top}(\vec{\sigma}_{S,t}^{-1})^{\top}\vec{\sigma}_{S,t}^{-1}\mathtt{I}_{ND}}, 0\right)$, ie.

$$\upsilon_t^{ND} = \begin{cases} \frac{-\sigma_{Y_h}\sigma_\alpha^2\left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y_h}\right]}{\left(\varphi\lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2\sigma_{Y_h}^2 + \sigma_\alpha^2} & \text{if } \Delta m_t^{Y_h} > \left(\varphi\left(1+\lambda_t\right) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Y_h}, \\ 0 & \text{otherwise.} \end{cases} \tag{46}$$

The collateral adjustment in this case is $\delta(\upsilon_t^{ND}) = -\varphi \upsilon_t^{ND}$.

A.1 r_t When Non-Domestic Holdings Are Constrained

$$r_t^{ND} = r_t^U + s_f^F \sigma_{Yf} (\vec{\sigma}_{S,t}^{-1} \upsilon_t^{ND})_{\text{el.2}} - s_f^F \delta(\upsilon_t^{ND}) \tag{47}$$

where $(\cdot)_{\text{el.i}}$ denotes the i'th element of the vector $(\cdot)$.

In scenario ND, $(\vec{\sigma}_{S,t}^{-1} \upsilon_t^{ND})_{\text{el.2}} = 0$. Being restricted only in one particular asset, S_t^h, sufficient alternative securities remain such that the two investors can efficiently trade the other source of fundamental risk, the *foreign* production risk. Accordingly, investors' beliefs about this unrestricted market will be correctly reflected by portfolios: $\Delta\kappa_t^f = \Delta m_t^{Y_f}$. F reallocates part of his wealth into a combination of the *home* bond B_t^h, providing exposure to exchange rate risk that he would otherwise carry through the inaccessible stock S_t^h, and the remaining risky asset, S_t^f.

However, the collateral adjustment $\delta(\upsilon_t^{ND}) = \frac{\varphi\sigma_{Yh}\sigma_\alpha^2\left[\Delta m_t^{Y_h} - \left(\varphi(1+\lambda_t) - \alpha_t^H - (1-\alpha^F)\lambda_t\right)\sigma_{Yh}\right]}{\left(\varphi\lambda_t(\alpha_t^H + \alpha^F - 1) + (1-\alpha_t^H)(\alpha_t^H + (1-\alpha^F)\lambda_t)\right)^2 \sigma_{Yh}^2 + \sigma_\alpha^2} > 0$ affects the precautionary savings motive. A constraint on long positions implies that this (rather optimistic) investor would like to invest more and thus feels precluded from participating in future growth, which he partly compensates for by investing more of his wealth into the riskfree asset than would be the case for a standard agent of his utility; this lowers the interest rate.

Firstly, a constraint is tighter when the imposed investment limit, φ in case ND, is lowered: when the constraint binds, F is forced to liquidate part of his holdings of S_t^h. Secondly, a given constraint is tighter when it endogenously binds more severely: the constraint distorts the portfolio more, desired and realized portfolio are very different. This is the case when, for a fixed level of φ, investor F wants to holds a large long position in stock S_t^h. Investor beliefs and changes therein over time capture this latter effect. Higher $\Delta m_t^{Y_h}$ implies that investor F is more bullish about investment opportunities in *home* country, but cannot purchase more of the stock. So although holdings of the restricted stock are not explicitly affected, the constraint will feed back into other security markets.

First, consider a tightening of constraints in the latter sense: how is the interest rate affected when a constraint that is in place starts binding more severely. Recall from (**??**) that r_t^U is independent of beliefs regarding Y_t^h's growth rate; the riskfree asset provides one unit of the numeraire consumption good in the future, and is therefore independent of the consumption risks associated with other goods.

Substituting equilibrium terms from proposition 1 into (47) shows that this independence no longer holds when the *foreign* investor is bound by his constraint:

$$\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_h}} = \frac{-\varphi s_f^F \sigma_{Yh} \sigma_\alpha}{\left(\varphi \lambda_t (\alpha_t^H + \alpha^F - 1) + (1 - \alpha_t^H)(\alpha_t^H + (1 - \alpha^F)\lambda_t)\right)^2 \sigma_{Yh}^2 + \sigma_\alpha^2} < 0.$$

The interest rate r_t^{ND} falls as the *foreign* investor is constrained more severely. His portfolio reallocation increases demand for the riskfree bond, lowering interest rates.

Having three other assets available to compensate for the restriction of S_t^h holdings, the fundamental risk of the *foreign* country can be optimally shared among investors, in accordance with their true beliefs about growth rates. This implies that the risk-free rate's sensitivity to this risk is identical to that in an identical but unconstrained economy: $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Yf}} = \frac{\partial r_t^{U}}{\partial \Delta m_t^{Yf}} = s_f^F \sigma_{Yf} > 0$: as disagreement about the *foreign* growth rate increases, the aggregate expected growth rate of consumption increases, putting upwards pressure on interest rate.

The model does not make explicit assumptions about which country's border the constraints are imposed at. The restriction on S_t^h for investor F could be due to *home* keeping foreign investment out or indeed due to the *foreign* government attempting to keep money in the country. The former type of restrictions are more commonly considered when thinking of the liberalization efforts during the 1980's and 1990's. The latter however also exist, for example in Argentina during a period in the early 2000s, and China still retains some restrictions on capital leaving local markets.

Now consider the other notion of 'tightening' constraints: a regulatory decision to lower the level φ. In contrast to the situation where the investor was more severely affected by the restriction due to his beliefs, this type of regulatory change leads to trade in the restricted asset. Investor F, already constrained, now has to sell some of his holdings in the *home* stock and reallocate them elsewhere. The effect of this reallocation on interest rates is ambiguous:

$$\frac{\partial r_t^{ND}}{\partial \varphi}\begin{cases} >0 & \text{if } \varphi > \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \\ <0 & \text{if } \varphi < \sqrt{\frac{(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Yf}^2+\sigma_\alpha^2}{\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Yf}^2}} \ \& \\ & \Delta m_t^{Y_h} > \left(\varphi\,(1+\lambda_t)-\alpha_t^H-(1-\alpha^F)\lambda_t\right)\sigma_{Yh}+A(\varphi) \end{cases}$$

where $A(\varphi)$ is a positive term, the details of which can be found in the appendix.

Crucially, whether tightening investment restrictions has a positive or negative effect on interest rates itself depends on how severely it was binding at the time the change is implemented. This interaction effect has not been discussed in any detail within both the theoretical and the empirical literature.

Consider the first of these two cases above: when φ is relatively lenient, changing the regulation to make this limit of *home* stockholding more strict will have the intuitive effect: As the constraint is tightened and F is forced to liquidate some of his holdings of S_t^h, he reallocates some of these freed funds into substitute assets, including his local bond market B_t^f. The increased demand for bonds means markets will clear at lower interest rates.

Conversely, tightening the investment limit has the opposite effect on interest rates when the constraint is already quite strict (φ low), and it binds severely ($\Delta m_t^{Y_h}$ high). A very strict limit implies that to take on the desired amount of risk, F has to hold large positions in the substitute assets. These, however, are imperfectly correlated with S_t^h. The investor faces the trade off between sacrificing his total amount of risk exposure or the diversification in his portfolio.

The more severely the constraint binds, the more he is willing to sacrifice diversification in response to a sudden change in regulation. He will tilt his portfolio more towards the alternative risky assets, to retain sufficient exposure to economic risk. Participating in the (high) expected growth of the economy through sufficient risk exposure overrides a risk-averse agent's desire to hold a diversified portfolio. The resulting drop in F's demand for bonds raises interest rates.

This trade off made by the restricted investor between diversification and overall risk exposure is not unique to this particular type of constraint. The next section will show how this intuition plays out in the setting of a leverage constraint. It illustrates clearly that regulatory intervention, often sought at times of high disagreement and uncertainty, can have counterintuitive results which must be considered.

B Optimal Consumption

Investors H and F maximize their respective expected utility, subject to budget constraints. Equilibrium is established by maximizing the aggregated utility function

$$U(C_H, C_F) = u_H\left(C^h_{H,t}, C^f_{H,t}\right) + \lambda_t u_F\left(C^h_{F,t}, C^f_{F,t}\right)$$

where

$$\begin{aligned} u_H\left(C^h_{H,t}, C^f_{Ht}\right) &= \alpha^H_t \log C^h_{Ht} + (1-\alpha^H_t)\log C^f_{H,t}, \\ u_F\left(C^h_{F,t}, C^f_{Ft}\right) &= (1-\alpha^F)\log C^h_{Ft} + \alpha^F \log C^f_{F,t}, \end{aligned}$$

and $\lambda_t = \frac{y_H \xi^H_t}{y_F \xi^F_t}$, the ratio of investors' state price densities.

FOC of optimal consumption of goods $j = h, f$, of investors $i = H, F$: $u^i_{C^j}(\cdot) = \frac{\partial u_i\left(C^i_{it}, C^j_{it}\right)}{\partial C^j_{it}} = y_i p^j_t \xi^i_t$, where p^j_t is the relative price of good j, ξ^i_t is investor i's state price density and y_i the associated Lagrange multiplier, reflecting initial endowment.

	investor H:	*investor F:*
good h:	$\frac{\alpha^H_t}{C^h_{Ht}} = y_H p^h_t \xi^H_t$	$\frac{1-\alpha^F}{C^h_{Ft}} = y_F p^h_t \xi^F_t$
good f:	$\frac{1-\alpha^H_t}{C^f_{Ht}} = y_H p^f_t \xi^H_t$	$\frac{\alpha^F}{C^f_{Ft}} = y_F p^f_t \xi^F_t$

Market clearing requires $\sum_i C^j_i = Y^j$ for both goods $j = h, f$, giving equilibrium total consumption in section 4.

C Optimal Wealth

Current wealth is an appropriately discounted value of all future consumption levels. Log utility in a finite horizon economy implies that both investors will consume a fixed portion of their wealth each period, as a function of the time remaining. The below is described for investor H, analogous values for investor F follow directly.

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\xi_s^H p_s^h C_{Hs}^h + \xi_s^H p_s^f C_{Hs}^f\right) ds\right]$$

From FOC above, $\frac{\alpha_t^H}{y_H} = C_{Ht}^h p_t^h \xi_t^H$ and $\frac{1-\alpha_t^H}{y_H} = C_{Ht}^f p_t^f \xi_t^H$ holds, therefore:

$$X_t^H = \frac{1}{\xi_t^H} E\left[\int_t^T \left(\frac{\alpha_s^H}{y_H} + \frac{1-\alpha_s^H}{y_H}\right) ds\right] = \frac{1}{y_H \xi_t^H}(T-t).$$

Linking wealth X_t^i back to consumption above gives

$$\begin{aligned} X_t^H &= C_{Ht}^h \cdot \frac{p_t^h}{\alpha_t^H}(T-t) = C_{Ht}^f \cdot \frac{p_t^f}{1-\alpha_t^H}(T-t), \\ X_t^F &= C_{Ft}^h \cdot \frac{p_t^h}{1-\alpha^F}(T-t) = C_{Ft}^f \cdot \frac{p_t^f}{\alpha^F}(T-t). \end{aligned}$$

D Relative Goods Prices

The relative price of the two goods is determined by their relative marginal utilities, which must be equal across the two agents, since both are faced with identical prices for goods, there are no frictions in goods markets: $\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u_{C^f}^i(\cdot)}{u_{C^h}^i(\cdot)}$. The basket of goods $\beta p_t^h + (1-\beta)\, p_t^f = 1$ defines the numeraire. $\beta \in [0,1]$ and represents the weight of the *home* good in the basket. This weight does not represent either agent's de facto consumed basket. The levels of stock prices will be affected by the chosen β, but the relation between the two stocks will not be. Interesting special cases include $\beta = 0$, $\beta = 1$ or $\beta = \alpha^F$, denoting Y_t^f, Y_t^h or F's true consumption basket as the numeraire, respectively. The main insights from the paper are not sensitive to the choice of β.

Using the equilibrium marginal utilities from market clearing restrictions $\sum_i C_i^j = Y^j$ for goods

$j = h, f$ gives:

$$\bar{p}_t = \frac{p_t^f}{p_t^h} = \frac{u_{C^f}^H(\cdot)}{u_{C^h}^H(\cdot)} = \frac{y_H p_t^f \xi_t^H}{y_H p_t^h \xi_t^H} = \frac{(1-\alpha_t^H) + \alpha^F \lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{Y_t^f}.$$

The dynamics of relative goods prices $\bar{p}_t$ follow

$$\begin{aligned} d\bar{p}_t &= (\cdot)dt + \frac{1-\alpha_t^H + \alpha^F\lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{1}{Y_t^f} dY_t^h - \frac{1-\alpha_t^H + \alpha^F\lambda_t}{\alpha_t^H + (1-\alpha^F)\lambda_t} \frac{Y_t^h}{(Y_t^f)^2} dY_t^f - \\ &\quad - \frac{\lambda_t + 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\alpha_t^H + \frac{2\alpha_t^H - 1}{(\alpha_t^H + (1-\alpha^F)\lambda_t)^2} \frac{Y_t^h}{Y_t^f} d\lambda_t. \end{aligned}$$

E Auxiliary Market: Portfolio Choice in Constrained Markets

The constraints studied are limitations on the fraction of wealth $\pi_{i,t}^j$ that investor i places into one or more assets j. I assume that portfolio positions $\pi_{i,t}^j$ in assets $j = S_t^h, S_t^f, B_t^h, B_t^f$ are constrained to lie in a closed, convex, non-empty set K that contains the origin. The analysis here is based on the methodology developed in Cvitanic and Karatzas (1992).

The martingale analysis of incomplete markets requires the construction of a fictitious market that fictitiously augments the market parameters of the original constrained market. Under these augmented market parameters, the constrained investor will optimally choose a portfolio permissible within the constraints. This is then the optimal portfolio also under the original, constrained market.[25]

The set of admissible trading strategies is defined by the set K, the support function is $\delta(\upsilon_t^i) \equiv \delta(\upsilon_t^i|K) \equiv \sup\left(-\pi_{i,t}^\top \upsilon_t^i : \pi_{i,t} \in K\right)$ and the barrier cone of the set $-K$ is defined as $\bar{K} \equiv \left\{\upsilon_t^i \in \mathbb{R}^2 | \delta(\upsilon_t^i) < \infty\right\}$. υ_t^i is a square-integrable, progressively measurable process taking values in $\bar{K}$ to ensure boundedness.

Investor F's state price density adjust to reflect these augmented market perceptions due to the constraints:

$$d\xi_t^F = -\left(r_t + \delta(\upsilon_t^F)\right)\xi_t^F dt - \kappa_t^{F\top} \xi_t^F d\vec{W}_t^{(F)}, \tag{48}$$

[25]This setting is a straightforward application of that in Cvitanic and Karatzas (1992), and it can be easily shown that their convex duality approach for convex constraint sets holds here.

where investor F's adjusted market price of risk is $\vec{\kappa}_t^F = (\sigma_{S,t}^{-1})\left(m_{S,t}^{(F)} + \upsilon_t^F \iota_F - r_t \mathbf{1}\right) = \kappa_{o,t}^F + \sigma_{S,t}^{-1}\upsilon_t^F$. $\kappa_{o,t}^F$ represents the market price of risk that the investor would base his portfolio decisions on, i.e. those reflecting his true beliefs. The second term, $+\sigma_{S,t}^{-1}\upsilon_t^F$, adjusts the market price of risk s.t. the investor does not violate his constraint, and at the same time captures the market price of risk that will be reflected in portfolio choice and thus equilibrium market prices.

F State Price Density

Investor H consumes a fraction $\frac{\alpha_t^H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{1-\alpha_t^H}{1-\alpha_t^H + \alpha^F \lambda_t}$ of good Y_t^f. This and equilibrium relative prices $\bar{p}_t$ gives

$$\xi_t^H = \beta \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{y_H Y_t^h} + (1-\beta)\,\frac{1-\alpha_t^H + \alpha^F \lambda_t}{y_H Y_t^f}. \tag{49}$$

Analogously, investor F consumes a fraction $\frac{\lambda_t(1-\alpha^F)}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ of good Y_t^h and a fraction $\frac{\lambda_t \alpha^F}{1-\alpha_t^H + \alpha^F \lambda_t}$ of good Y_t^f:

$$\xi_t^F = \beta \frac{\alpha_t^H + (1-\alpha^F)\lambda_t}{\lambda_t y_F Y_t^h} + (1-\beta)\,\frac{1-\alpha_t^H + \alpha^F \lambda_t}{\lambda_t y_F Y_t^f}. \tag{50}$$

G Asset Valuation

Proof of Proposition 1: The proof follows closely that in Schornick (2009), under the simpler situation that H does not face a constraint.

Market clearing in asset markets requires

$$S_t^h + S_t^f = X_t^H + X_t^F = p_t^h Y_t^h (T-t) + p_t^f Y_t^f (T-t). \tag{51}$$

Each asset $j = h, f$ is valued as the sum of discounted dividends, taking into account the effects of future binding constraints — the second integral in the equation below.

$$S_t^j = \frac{1}{\xi_t^H} E_t\left[\int_t^T \xi_s^H p_s^j Y_s^j \, ds\right] \qquad j = h, f.$$

Using $\frac{1}{p_t^h \xi_t^H} = \frac{Y_t^h y_H}{\alpha_t^H + (1-\alpha^F)\lambda_t}$ and goods market clearing, as well as $\lambda_t = \frac{y_H \xi_t^H}{y_F \xi_t^F}$ in the pricing function of S_t^h:

$$S_t^h = p_t^h Y_t^h (T-t) + \frac{p_t^h Y_t^h}{\alpha_t^H + (1-\alpha^F)\lambda_t}(1-\alpha^F)\left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{52}$$

$$S_t^f = p_t^f Y_t^f (T-t) + \frac{p_t^f Y_t^f}{1-\alpha_t^1 + \alpha^2 \lambda_t}\alpha^F \left[E_t \int_t^T \lambda_s ds - \lambda_t (T-t)\right] \tag{53}$$

Under the constraints on investor F $d\lambda_t$ is a supermartingale under all possible equilibria. Therefore,

$$\begin{aligned} S_t^h &= p_t^h Y_t^h (T-t), \\ S_t^f &= p_t^f Y_t^f (T-t), \end{aligned} \tag{54}$$

where p_t^h and p_t^f can be rewritten in terms of $\bar{p}_t$.

H Interest Rate Effects

In section III.A the sensitivity of the interest rate in scenario ND with respect to restriction parameter φ is detailed. $A(\varphi) = \frac{(1+\lambda_t)\varphi\sigma_{Y_h}\left(\left(\varphi\lambda_t(\alpha_t^H+\alpha^F-1)+(1-\alpha_t^H)(\alpha_t^H+(1-\alpha^F)\lambda_t)\right)^2\sigma_{Y_h}^2+\sigma_\alpha^2\right)}{-\varphi^2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_h}^2+(1-\alpha_t^H)^2(\alpha_t^H+(1-\alpha^F)\lambda_t)^2\sigma_{Y_h}^2+\sigma_\alpha^2} > 0$

In scenario LC, the function that determines the sign of $\frac{\partial r_t^{ND}}{\partial \Delta m_t^{Y_f}}$ is

$$B(\cdot) = \frac{\sigma_\alpha^2\sigma_{Y_h}^2 \pm \sqrt{\left(\sigma_\alpha^2\sigma_{Y_h}^2+4\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2(\sigma_{Y_h}^2-\sigma_{Y_f}^2)\right)\sigma_\alpha^2\sigma_{Y_h}^2}}{2\lambda_t^2(\alpha_t^H+\alpha^F-1)^2\sigma_{Y_f}^2\sigma_{Y_h}^2}.$$

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